results given in the literature are in good agreement with the results obtained by the method described here. Closed-form solutions are given for some stability diagrams. These were obtained in the literature with the help of computers.

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Behavior of a Two Degree-of-Freedom Gyroscope in a Rotating Satellite

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The equations of motion of a two degree-of-freedom gyroscope mounted in an artificial satellite are used to formulate conditions under which the spin-axis of the rotor can remain nearly fixed in inertial space. As an illustrative application, an attitude control scheme for Earth-pointing satellites is devised.

Nomenclature

a	= distance from Q to T
$\mathbf{a}_1$	= unit vector parallel to QT
$\mathbf{a}_2$	$= \mathbf{a}_3 \times \mathbf{a}_1$
$\mathbf{a}_3$	= unit vector parallel to E_3
b	= distance from P to T
$\mathbf{b}_i (i = 1,2,3)$	= unit vectors parallel to principal axes of inertia of B for P
$B_i (i = 1,2,3)$	= principal moments of inertia of B for P
$\mathbf{c}_i (i = 1,2,3)$	= unit vectors parallel to principal axes of inertia of C for P
$C_i (i = 1,2,3)$	= principal moments of inertia of C for P
D_1	= moment of inertia of D about any line passing through P and perpendicular to $\mathbf{c}_2$
D_2	= moment of inertia of D about symmetry axis
$\mathbf{e}_i (i = 1,2,3)$	= unit vector parallel to E_i
R	= radius of circular orbit
α	= angle between $\mathbf{e}_1$ and $\mathbf{a}_1$
β	= angle between $\mathbf{a}_1$ and $\mathbf{b}_1$
γ	= angle between $\mathbf{b}_2$ and $\mathbf{c}_2$
σ	= angular speed of D relative to C
Ω	= angular speed of line $O - Q$ in an inertial reference frame

Introduction

TWO degree-of-freedom gyroscopes are employed extensively in inertial navigation and guidance systems. It has been shown that the spin-axis of such a gyroscope does not necessarily have a fixed orientation in inertial space when the gyroscope is mounted in a rotating vehicle.[1] Hence, it is important that vehicle motions be taken into account when dealing with such systems.

It is the purpose of this paper to show how one can cause the spin-axis of a two degree-of-freedom gyroscope that is mounted in a rotating satellite to remain nearly fixed in inertial space. The results obtained are used to devise an attitude control scheme for Earth-pointing satellites, and it is demonstrated that this scheme leads to significant improvements in roll and yaw control.

System Description

In Fig. 1, O designates a particle fixed in an inertial reference frame, and A is an artificial satellite whose mass center, Q, moves with constant angular speed in a circular orbit

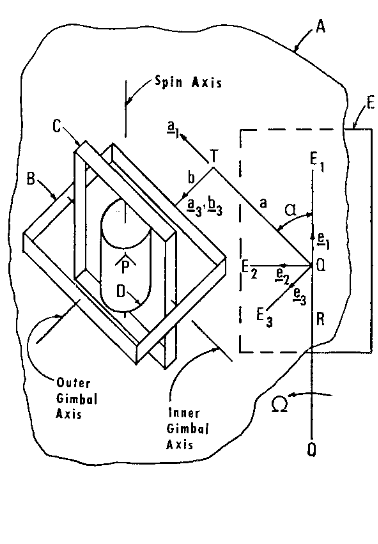

Fig. 1 Satellite and gyroscope.

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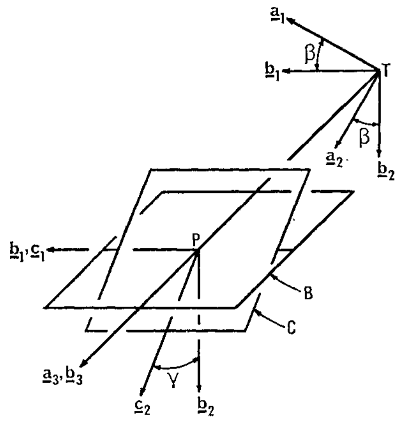

Fig. 2 Gimbal angles.

centered at O. The system to be analyzed is a gyroscope consisting of an outer gimbal B, an inner gimbal C, and a rotor D.

To describe rotational motions of A, we introduce an orbiting reference frame E, in which mutually perpendicular axes E_1, and E_2, and E_3 are fixed, E_1 being the extension of line OQ, E_2 the tangent to the circular orbit at Q, and E_3 the normal to the orbit plane; and we restrict the motion of A in E to rotation about E_3. As for the gyroscope, the outer gimbal axis is parallel to E_3, and the mass centers of B, C, and D coincide at point P. The rotor is driven at a constant angular speed relative to C, and all gimbals are presumed to be frictionless. T is the point of intersection of the outer gimbal axis and the orbit plane.

Unit vectors and angles used to describe the relative orientations of A,B,C,D, and E are shown in Figs. 1 and 2 and are listed in the Nomenclature, as are dimensions and inertia properties of interest.

Equation of Motion

The differential equations governing β and γ are obtained by forming the generalized forces F_β and F_γ and the total kinetic energy K and then using the Lagrange equations

$$(d/dt)\partial K/\partial\dot\beta - \partial K/\partial\beta = F_\beta$$
$$(d/dt)\partial K/\partial\dot\gamma - \partial K/\partial\gamma = F_\gamma \qquad (1)$$

Regarding the gravitational forces of interaction between A, B, C, and D as negligible, and replacing the systems of gravitational forces exerted by O on B, C, and D with forces $\mathbf{F}^B$, $\mathbf{F}^C$, and $\mathbf{F}^D$ applied at P together with couples of torques $\mathbf{T}^B$, $\mathbf{T}^C$, and $\mathbf{T}^D$, one[2] can express the latter with sufficient accuracy as

$$\mathbf{T}^B = 3GM(\mathbf{r}^2)^{-5/2}\mathbf{r}\times\mathbf{I}^B\cdot\mathbf{r} \qquad (2a)$$
$$\mathbf{T}^C = 3GM(\mathbf{r}^2)^{-5/2}\mathbf{r}\times\mathbf{I}^C\cdot\mathbf{r} \qquad (2b)$$
$$\mathbf{T}^D = 3GM(\mathbf{r}^2)^{-5/2}\mathbf{r}\times\mathbf{I}^D\cdot\mathbf{r} \qquad (2c)$$

where G is the universal gravitational constant, M is the mass of O, $\mathbf{r}$ is the position vector of P relative to O, and $\mathbf{I}^B$, $\mathbf{I}^C$, and $\mathbf{I}^D$ are the inertia dyadics of B,C, and D for P. If a and b are small compared to R, F_β and F_γ are then given by

$$F_\beta = (3GM/2R^3)[B_1 + C_1 - B_2 - C_3 + (C_3 + D_1 - C_2 - D_2)\cos^2\gamma]\sin 2(\beta+\alpha) \qquad (3)$$
$$F_\gamma = (3GM/2R^3)(C_2 + D_2 - C_3 - D_1)\sin^2(\beta+\alpha)\sin 2\gamma$$

Confining attention to situations in which A rotates with constant (positive, negative, or zero) angular speed in E, so that α can be expressed as $\alpha = \alpha_O + \omega t$, where α_O and ω are constants, and using the fact that $GM = \Omega^2 R^3$, one arrives at the following differential equations governing β and γ:

$$[B_3 + C_2 + D_2 + (C_3 + D_1 - C_2 - D_2)\cos^2\gamma]\ddot\beta + D_2\sigma\dot\gamma\cos\gamma - (C_3 + D_1 - C_2 - D_2)\times(\Omega+\omega+\dot\beta)\dot\gamma\sin 2\gamma - \tfrac{3}{2}[B_1 + C_1 - B_2 - C_3 + (C_3 + D_1 - C_2 - D_2)\cos^2\gamma]\Omega^2\sin 2(\beta+\omega t+\alpha_O) = 0 \qquad (4)$$
$$(C_1 + D_1)\ddot\gamma - D_2(\Omega+\omega+\dot\beta)\sigma\cos\gamma + (C_3 + D_1 - C_2 - D_2)(\Omega+\omega+\dot\beta)^2\sin\gamma\cos\gamma + \tfrac{3}{2}(C_3 + D_1 - C_2 - D_2)\Omega^2\sin^2(\beta+\omega t+\alpha_O)\sin 2\gamma = 0$$

These equations can be cast into a more convenient form after defining $J_1, \ldots, J_6$ as

$$J_1 = B_3 + C_2 + D_2;\; J_2 = C_3 + D_1 - C_2 - D_2;\; J_3 = D_2;$$
$$J_4 = B_1 + C_1 - B_2 - C_3;\; J_5 = C_1 + D_1; \qquad (5)$$
$$J_6 = C_3 + D_1 - C_2 - D_2$$

Substitution from Eqs. (5) into Eqs. (4) then yields

$$(J_1 + J_2\cos^2\gamma)\ddot\beta + J_3\sigma\dot\gamma\cos\gamma - J_2(\Omega+\omega+\dot\beta)\dot\gamma\sin 2\gamma - \tfrac{3}{2}(J_4 + J_6\cos^2\gamma)\Omega^2\sin 2(\beta+\omega t+\alpha_O) = 0 \qquad (6)$$
$$J_5\ddot\gamma - J_3(\Omega+\omega+\dot\beta)\sigma\cos\gamma + \tfrac{1}{2}J_2(\gamma+\omega+\dot\beta)^2\sin 2\gamma + \tfrac{3}{2}J_6\Omega^2\sin^2(\beta+\omega t+\alpha_O)\sin 2\gamma = 0$$

It is worth noting that Eqs. (6) can be shown to govern the behavior of a gyroscope with a free rather than a driven rotor if the definition of J_2 [see Eqs (5)] is changed to $J_2 = C_3 + D_1 - C_2$, and if σ is eliminated by using the relationship

$$\sigma = s^* + (\Omega+\omega+\dot\beta^*)\sin\gamma^* \qquad (7)$$

where s^*, $\dot\beta^*$, and γ^* are, respectively, the values of the angular speed of D in C, $\dot\beta$, and γ at some particular instant of time.

Spin-Axis Behavior

In the sequel we will be concerned mainly with the motion of the spin-axis of the rotor. Hence, it is useful to express a unit vector parallel to the spin-axis in terms of unit vectors fixed in inertial space.

Letting $\mathbf{e}_i$ be a unit vector parallel to E_i $(i=1,2,3)$ and defining $\mathbf{n}_i$ as a unit vector fixed in inertial space in such a way that, at $t=0$, $\mathbf{n}_i = \mathbf{e}_i$ $(i=1,2,3)$, one finds that the unit vector $\mathbf{c}_2$, which is parallel to the spin-axis, can be expressed as

$$\mathbf{c}_2 = -\cos\gamma\sin[\beta + (\Omega+\omega)t + \alpha_O]\mathbf{n}_1 + \cos\gamma\cos[\beta + (\Omega+\omega)t + \alpha_O]\mathbf{n}_2 + \sin\gamma\mathbf{n}_3 \qquad (8)$$

Alternatively, denoting by $\mathbf{c}_{iO}$ a unit vector fixed in inertial space and equal to $\mathbf{c}_i$ $(i=1,2,3)$ at $t=0$, one can write

$$\mathbf{c}_2 = x\mathbf{c}_{10} + y\mathbf{c}_{20} + z\mathbf{c}_{30} \qquad (9)$$

From Eqs. (8) and (9) it then follows that

$$x = -\cos\gamma\sin[\beta + (\Omega+\omega)t - \beta_O] \qquad (10a)$$
$$y = \sin\gamma\sin\gamma_O + \cos\gamma\cos\gamma_O\cos[\beta + (\Omega+\omega)t - \beta_O] \qquad (10b)$$
$$z = \sin\gamma\cos\gamma_O - \cos\gamma\sin\gamma_O\cos[\beta + (\Omega+\omega)t - \beta_O] \qquad (10c)$$

The quantities x and z characterize the behavior of the spin-axis and are referred to as "drifts."

Spin-Axis Nearly Fixed in Inertial Space

A glance at Eqs. (6) reveals that $\beta = -\omega t - \alpha_O + m\pi/2$ $(m=0,1)$ and $\gamma = \pi/2$ constitute exact solutions. The associated value of $\mathbf{c}_2$ is

$$\mathbf{c}_2 = \mathbf{n}_3 \qquad (11)$$

which means that the spin-axis remains normal to the orbit plane (and thus fixed in inertial space). It will now be shown that, under certain conditions, the spin-axis can remain nearly fixed in certain directions not normal to the orbit plane. Expressions describing the behavior of β and γ during the associated motion will be obtained by an iterative procedure.

When terms due to gravity torque are omitted from Eqs. (6), these equations reduce to

$$(J_1 + J_2 \cos^2\gamma)\ddot{\beta} + J_3\sigma\dot{\gamma}\cos\gamma - J_2(\Omega + \omega + \dot{\beta})\dot{\gamma}\sin 2\gamma = 0$$
$$J_5\ddot{\gamma} - J_3(\Omega + \omega + \dot{\beta})\sigma\cos\gamma + \tfrac{1}{2}J_2(\Omega + \omega + \dot{\beta})^2 \sin 2\gamma = 0 \tag{12}$$

which are satisfied by $\beta = -(\Omega + \omega)t + \beta_O$ and $\gamma = \gamma_O$, where β_O and γ_O are the values of β and γ at $t = 0$. When β and γ are given by these expressions, the spin-axis remains fixed in inertial space in an orientation depending solely on β_O and γ_O; that is

$$\mathbf{c}_2 = -\cos\gamma_O \sin(\beta_O + \alpha_O)\mathbf{n}_1 + \cos\gamma_O\cos(\beta_O + \alpha_O)\mathbf{n}_2 + \sin\gamma_O\mathbf{n}_3 \tag{13}$$

It is worth noting that $\dot{\beta}_O$ and $\dot{\gamma}_O$, the associated initial values of $\dot{\beta}$ and $\dot{\gamma}$, cannot be chosen arbitrarily. In fact, $\dot{\beta}_O = -(\Omega + \omega)$ and $\dot{\gamma}_O = 0$.

We now use $\beta = -(\Omega + \omega)t + \beta_O$ and $\gamma = \gamma_O$ to express as explicit functions of time the terms associated with gravity torque in Eqs. (6), which gives

$$(J_1 + J_2\cos^2\gamma)\ddot{\beta} + J_3\sigma\dot{\gamma}\cos\gamma - J_2(\Omega + \omega + \dot{\beta})\dot{\gamma}\sin 2\gamma = \tfrac{3}{2}(J_4 + J_6\cos^2\gamma_O)\Omega^2 \sin 2(-\Omega t + \beta_O + \alpha_O) \tag{14}$$
$$J_5\ddot{\gamma} - J_3(\Omega + \omega + \dot{\beta})\sigma\cos\gamma + \tfrac{1}{2}J_2(\Omega + \omega + \dot{\beta})^2\sin 2\gamma = -\tfrac{3}{2}J_6\Omega^2\sin 2\gamma_O \sin^2(-\Omega t + \beta_O + \alpha_O)$$

Integrating the first of Eqs. (14) once and making use of $\dot{\beta}_O = -(\Omega + \omega)$, one finds that

$$\dot{\beta} = -(\Omega + \omega) + \frac{h_1 - J_3\sigma\sin\gamma}{J_1 + J_2\cos^2\gamma} + \frac{3}{2}\frac{J_4 + J_6\cos^2\gamma_O}{J_1 + J_2\cos^2\gamma}\Omega\cos^2(-\Omega t + \beta_O + \alpha_O) \tag{15}$$

where h_1 is a constant defined as

$$h_1 = J_3\sigma\sin\gamma_O - \tfrac{3}{2}(J_4 + J_6\cos^2\gamma_O)\Omega\cos 2(\beta_O + \alpha_O) \tag{16}$$

Next, assuming that γ can be expressed as $\gamma = \gamma_O + q$ where q is so small that only terms linear in q need to be retained, one arrives by substitution into Eq. (15) at

$$\dot{\beta} = -(\Omega + \omega) + h_2 + h_3\Omega\cos 2(-\Omega t + \beta_O + \alpha_O) + h_4 q \tag{17}$$

where h_2, h_3, and h_4 are constants given by

$$h_2 = \frac{h_1 - J_3\sigma\sin\gamma_O}{J_1 + J_2\cos^2\gamma_O}, \quad h_3 = \frac{3}{2}\frac{J_4 + J_6\cos^2\gamma_O}{J_1 + J_2\cos^2\gamma_O}$$
$$h_4 = -\frac{J_3\sigma\cos\gamma_O}{J_1 + J_2\cos^2\gamma_O} + \frac{J_2\sin 2\gamma_O(h_1 - J_3\sigma\sin\gamma_O)}{(J_1 + J_2\cos^2\gamma_O)^2} \tag{18}$$

Moreover, q must satisfy the differential equation obtained by substituting $\gamma = \gamma_O + q$ and $\dot{\beta}$, as given by Eq. (17), into the second of Eqs. (14) and linearizing in q, which leads to

$$\ddot{q} + p^2 q = h_5 + h_6\cos 2(-\Omega t + \beta_O + \alpha_O) + h_7\cos 4(-\Omega t + \beta_O + \alpha_O) \tag{19}$$

where p^2, h_5, h_6, and h_7 are constants defined as

$$p^2 = (1/J_5)[J_3 h_2\sigma\sin\gamma_O - J_3 h_4\sigma\cos\gamma_O + J_2 h_2 h_4\sin 2\gamma_O + J_2(h_2{}^2 + \tfrac{1}{2}h_3{}^2\Omega^2)\cos 2\gamma_O] \tag{20a}$$
$$h_5 = (1/J_5)[J_3 h_2\sigma\cos\gamma_O - \tfrac{3}{4}J_6\Omega^2\sin 2\gamma_O - \tfrac{1}{4}J_2(h_3{}^2\Omega^2 + 2h_2{}^2)\sin 2\gamma_O] \tag{20b}$$
$$h_6 = (1/J_5)[J_3 h_3\sigma\Omega\cos\gamma_O + \tfrac{3}{4}J_6\Omega^2\sin 2\gamma_O - J_2 h_2 h_3\Omega\sin 2\gamma_O \tag{20c}$$
$$h_7 = -\tfrac{1}{4}(J_2/J_5)h_3{}^2\Omega^2\sin 2\gamma_O \tag{20d}$$

The solution of Eq. (19) satisfying the conditions $q = \dot{q} = 0$ at $t = 0$ (recall that $\gamma = \gamma_O + q$ and $\dot{\gamma}_O = 0$) is

$$q = k_1\cos pt + k_2\sin pt + h_5/p^2 + [h_6/(p^2 - 4\Omega^2)]\cos 2(-\Omega t + \beta_O + \alpha_O) + [h_7/(p^2 - 16\Omega^2)]\cos 4(-\Omega t + \beta_O + \alpha_O) \tag{21}$$

where k_1 and k_2 are defined as

$$k_1 = -(h_5/p^2) - [h_6/(p^2 - 4\Omega^2)]\cos 2(\beta_O + \alpha_O) - [h_7/(p^2 - 16\Omega^2)]\cos 4(\beta_O + \alpha_O)$$
$$k_2 = -(2\Omega/p)[h_6/(p^2 - 4\Omega^2)]\sin 2(\beta_O + \alpha_O) - (4\Omega/p)[h_7/(p^2 - 16\Omega^2)]\sin 4(\beta_O + \alpha_O) \tag{22}$$

Substituting q as given in Eq. (21) into Eq. (17), one obtains

$$= -(\Omega + \omega) + h_2 + (h_4 h_5/p^2) + h_4 k_1\cos pt + h_4 k_2\sin pt + \{h_3\Omega + [h_4 h_6/(p^2 - 4\Omega^2)]\}\cos 2(-\Omega t + \beta_O + \alpha_O) + [h_4 h_7/(p^2 - 16\Omega^2)]\cos 4(-\Omega t + \beta_O + \alpha_O) \tag{23}$$

and integration yields

$$\beta = \beta_O + [-(\Omega + \omega) + h_2 + h_4 h_5/p^2]t + (h_4 k_1/p)\sin pt - (h_4 k_2/p)(\cos pt - 1) - (1/2\Omega)\{h_3\Omega + [h_4 h_6/(p^2 - 4\Omega^2)]\}[\sin 2(-\Omega t + \beta_O + \alpha_O) - \sin 2(\beta_O + \alpha_O)] - (1/4\Omega)[h_4 h_7/(p^2 - 16\Omega^2)] \times [\sin 4(-\Omega t + \beta_O + \alpha_O) - \sin 4(\beta_O + \alpha_O)] \tag{24}$$

Similarly,

$$\gamma = \gamma_O + h_5/p^2 + k_1\cos pt + k_2\sin pt + [h_6/(p^2 - 4\Omega^2)]\cos 2(-\Omega t + \beta_O + \alpha_O) + [h_7/(p^2 - 16\Omega^2)]\cos 4(-\Omega t + \beta_O + \alpha_O) \tag{25}$$

Finally, defining Δ, ρ_1, and ρ_2 as

$$\Delta = h_2 + h_4 h_5/p^2 \tag{26}$$

$$\rho_1 = h_4 k_2/p + (1/2\Omega)\{(h_3\Omega + [h_4 h_6/(p^2 - 4\Omega^2)]\} \times \sin 2(\beta_O + \alpha_O) + (1/4\Omega)[h_4 h_6/(p^2 - 16\Omega^2)]\sin 4(\beta_O + \alpha_O) + (h_4 k_1/p)\sin pt - (h_4 k_2/p)\cos pt - (1/2\Omega)\{(h_3 + [h_4 h_6/(p^2 - 4\Omega^2)]\}\sin 2(-\Omega t + \beta_O + \alpha_O) - (1/4\Omega)[h_4 h_7/(p^2 - 16\Omega^2)]\sin 4(-\Omega t + \beta_O + \alpha_O)$$
$$\rho_2 = h_5/p^2 + k_1\cos pt + k_2\sin pt + [h_6/(p^2 - 4\Omega^2)]\cos 2(-\Omega t + \beta_O\alpha_O) + [h_7/(p^2 - 16\Omega^2)]\cos 4(-\Omega t + \beta_O + \alpha_O) \tag{27}$$

one obtains

$$\beta = \beta_O - (\Omega + \omega)t + \Delta t + \rho_1 \tag{28}$$

and

$$\gamma = \gamma_O + \rho_2 \tag{29}$$

Equation (28) shows that β will acquire values differing appreciably from $\beta_O - (\Omega + \omega)t$, unless $\Delta = 0$. To determine the conditions under which this requirement is satisfied, we express Δ in terms of inertia properties and the angular speeds Ω and σ [see Eqs. (18), (20), and (26)]:

$$\Delta = (\Omega^2/p^2)(\nu_1\sigma + \nu_2\Omega) \tag{30}$$

where ν_1 and ν_2 are constants defined as

$$\nu_1 = \frac{9}{16}\frac{J_1 J_3}{J_5}\frac{(J_4 + J_6\cos^2\gamma_O)^2}{(J_1 + J_2\cos^2\gamma_O)^3}\sin\gamma_O\cos^2 2(\beta_O + \alpha_O) + \frac{3J_6}{2} + \frac{9J_2}{32}\frac{(J_4 + J_6\cos^2\gamma_O)^2}{(J_1 + J_2\cos^2\gamma_O)^2}\frac{J_3}{J_5}\frac{\sin\gamma_O\cos^2\gamma_O}{J_1 + J_2\cos^2\gamma_O}$$
$$\nu_2 = \frac{9}{16}\frac{J_2}{J_5}\frac{J_4 + J_6\cos^2\gamma_O}{(J_1 + J_2\cos^2\gamma_O)^2}\cos 2(\beta_O + \alpha_O)J_6\left\{\sin^2 2\gamma_O - \frac{3}{16}J_2\frac{(J_4 + J_6\cos^2\gamma_O)^2}{(J_1 + J_2\cos^2\gamma_O)^2}\sin^2 2\gamma_O\cos 4(\beta_O + \alpha_O) - \frac{3}{8}\frac{(J_4 + J_6\cos^2\gamma_O)^2}{J_1 + J_2\cos^2\gamma_O}\cos 2\gamma_O[1 + 2\cos^2 2(\beta_O + \alpha_O)]\right\} \tag{31}$$

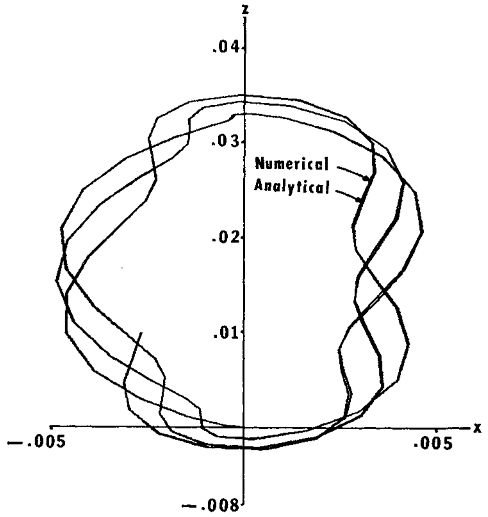

Fig. 3 Numerical and analytical solutions, $\Delta = 0$.

The requirement $\Delta = 0$ is seen to be satisfied whenever $\nu_1\sigma + \nu_2\Omega = 0$, which can be accomplished by taking $\sigma = -(\nu_2/\nu_1)\Omega$ or by choosing $J_2 = 0$ and either taking $\gamma_O = 0$ or $\beta_O + \alpha_O = \pi/4$, in which case $\nu_1 = \nu_2 = 0$, as may be seen by reference to Eqs. (31).

The validity of Eqs. (28) and (29) can be tested by using these equations, on the one hand, and a numerical integration of Eqs. (16), on the other hand, to evaluate β and γ. Plots of z vs x [see Eqs. (10)] constructed by using values of β and γ obtained in these two ways then furnish a convenient basis for comparison. Figures 3 and 4 show such plots for $\Delta = 0$ and $\Delta \neq 0$, respectively, for a gyroscope having the following moments of inertia (all expressed in slug-ft^2):

$$\begin{aligned} B_1 &= B_3 = 0.075, & B_2 &= 0.1 \\ C_1 &= C_2 = 0.05, & C_3 &= 0.075 \\ D_1 &= 0.205, & D_2 &= 0.23 \end{aligned} \qquad (32)$$

The rotor is presumed to be driven at constant rate, with $\sigma/\Omega = 10$, and the initial values of α, β, and γ are $\alpha_O = \beta_O = 0$, and, for $\Delta = 0$, $\gamma_O = 0$, whereas, for $\Delta \neq 0$, $\gamma_O = \pi/4$.

The trajectories in Fig. 3 correspond to values of Ωt between 0 and 10, whereas those in Fig. 4 apply for Ωt between 0 and 3. In Fig. 3, the agreement between the analytical and the numerical solution is seen to be so good that the associated curves are nearly indistinguishable. (Computations for values of Ωt up to 100 give equally good agreement.) Figure 4 indicates that, for $\Delta \neq 0$, the analytical solution gives correct results only initially. While this is not surprising, for the term Δt in Eq. (28) represents an unbounded departure of β from the nominal value used to generate the analytical solution, it gives rise to the following question: what is it that really happens if $\Delta \neq 0$? More particularly, do departures of the spin-axis from its original orientation become greater if $\Delta \neq 0$ than they do if Δ vanishes? This question can be answered most incisively by using a numerical solution of Eqs. (6) together with the second of Eqs. (10) to evaluate the angle ϕ defined as [see Eq. (9) for $\mathbf{c}_2$ and $\mathbf{c}_{20}$]

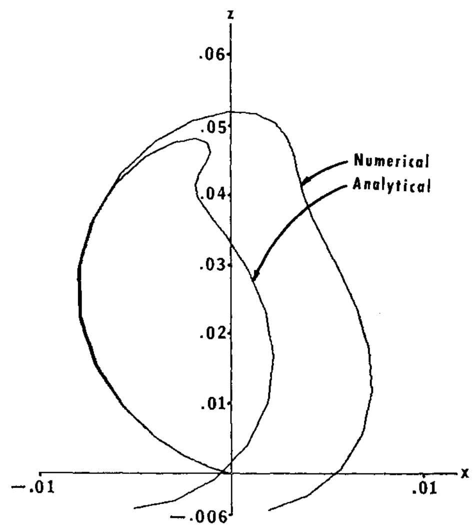

Fig. 4 Numerical and analytical solutions, $\Delta \neq 0$.

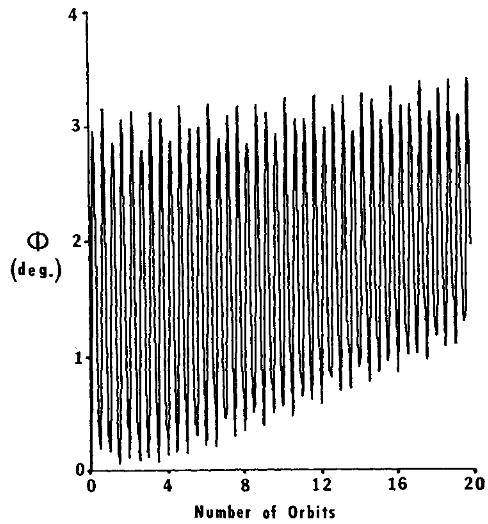

Fig. 5 Angle between $\mathbf{c}_2$ and $\mathbf{c}_{20}$, $\Delta \neq 0$.

$$\phi = \cos^{-1}(\mathbf{c}_2 \cdot \mathbf{c}_{20}) = \cos^{-1} y \qquad (33)$$

and then plotting ϕ vs the number of orbits traversed by the satellite. Figures 5 and 6 contain two such plots for the gyroscope and for the initial conditions considered previously. From these figures it is evident that $\Delta \neq 0$ indeed leads to a drift and that this drift can be eliminated by making $\Delta = 0$.

The results just obtained indicate both that it is practically advantageous to make $\Delta = 0$ and that Eqs. (28) and (29) then can be used with considerable confidence. Accordingly, we shall henceforth confine attention to arrangements for which $\Delta = 0$.

In the sequel, an expression for $\mathbf{c}_2$ as a function of time will be required. Taking advantage of the fact that ρ_1 and ρ_2 may be presumed to be small, one finds from Eqs. (8), (28), and (29) that

$$\begin{aligned} \mathbf{c}_2 = \mathbf{c}_{20} + [-\rho_1 \cos\gamma_O \cos(\beta_O + \alpha_O) + \\ \rho_2 \sin\gamma_O \sin(\beta_O + \alpha_O)]\mathbf{n}_1 - [\rho_1 \cos\gamma_O \sin(\beta_O + \alpha_O) + \\ \rho_2 \sin\gamma_O \cos(\beta_O + \alpha_O)]\mathbf{n}_2 + \rho_2 \cos\gamma_O \mathbf{n}_3 \end{aligned} \qquad (34)$$

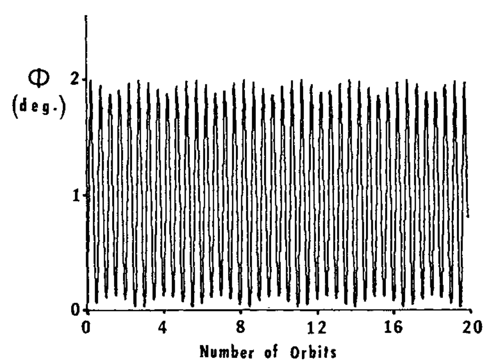

Fig. 6 Angle between $\mathbf{c}_2$ and $\mathbf{c}_{20}$, $\Delta = 0$.

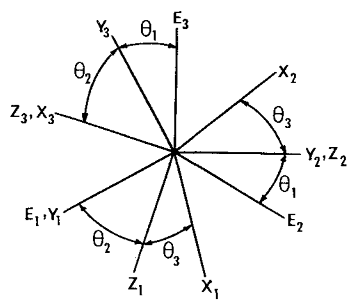

Fig. 7 Attitude angles.

Attitude Control of Earth-Pointing Satellites

The main result of the previous section, namely that the spin axis can be made to remain nearly fixed in inertial space, will now be used for attitude control of an Earth-pointing satellite. This is accomplished by using two gyroscopes to determine approximate values of the time derivatives of certain angles that describe the orientation of the satellite; and a torque proportional to these derivatives is then applied to the satellite to drive it to the desired orientation.

Let X_1, X_2 and X_3 be the principal axes of inertia of the satellite A for the mass center Q of A. The attitude of A relative to the orbiting reference frame E can be specified in terms of three angles θ_1, θ_2, and θ_3, generated as follows: align X_i with E_i, $i = 1,2,3$; perform a right-handed rotation of A of amount θ_1 about E_1, bringing X_i into coincidence with Y_i, $i = 1,2,3$ (see Fig. 7); and follow this with rotations of amount θ_2 about Y_2 leading to Z_1, Z_2, Z_3, and θ_3 about Z_3, bringing A into its final orientation.

Let $\mathbf{a}_i$ be a unit vector parallel to X_i, $i = 1,2,3$, and assume that a control torque $\mathbf{T}_c$ is applied to the satellite, $\mathbf{T}_c$ being given by

$$\mathbf{T}_c = -k(I_1\dot{\bar\theta}_1\mathbf{a}_1 + I_2\dot{\bar\theta}_2\mathbf{a}_2 + I_3\dot{\bar\theta}_3\mathbf{a}_3) \tag{35}$$

where I_1, I_2 and I_3 are the principal moments of inertia of A, k is a positive constant, and $\dot{\bar\theta}_1$, $\dot{\bar\theta}_2$, $\dot{\bar\theta}_3$ are approximate values of the time derivatives of θ_1, θ_2, θ_3, determined by using two gyroscopes in a manner to be described presently.

Denoting by ω_1, ω_2, ω_3 angular velocity components of A (in inertial space) referred to the axes X_1, X_2, X_3 one finds[3] that

$$\omega_1 = (\dot\theta_2 + \Omega\sin\theta_1)\sin\theta_3 + (\dot\theta_1\cos\theta_2 - \Omega\cos\theta_1\sin\theta_2)\cos\theta_3 \tag{36a}$$

$$\omega_2 = (\dot\theta_2 + \Omega\sin\theta_1)\cos\theta_3 - (\dot\theta_1\cos\theta_2 - \Omega\cos\theta_1\sin\theta_2)\sin\theta_3 \tag{36b}$$

$$\omega_3 = \dot\theta_1\sin\theta_2 + \Omega\cos\theta_1\cos\theta_2 + \dot\theta_3 \tag{36c}$$

Furthermore, ω_1, ω_2, and ω_3 must satisfy the dynamical equations

$$I_1\dot\omega_1 + (I_3 - I_2)\omega_2\omega_3 = M_1 - kI_1\dot{\bar\theta}_1 \tag{37a}$$

$$I_2\dot\omega_2 + (I_1 - I_3)\omega_3\omega_1 = M_2 - kI_2\dot{\bar\theta}_2 \tag{37b}$$

$$I_3\dot\omega_3 + (I_2 - I_1)\omega_1\omega_2 = M_3 - kI_3\dot{\bar\theta}_3 \tag{37c}$$

where M_1, M_2, and M_3, the components of the gravitational torque exerted on A by O referred to X_1, X_2, and X_3, are given by[4]

$$M_1 = (3GM/R^3)(I_2 - I_3)\sin\theta_2\cos\theta_2\sin\theta_3 \tag{38a}$$

$$M_2 = (3GM/R^3)(I_1 - I_3)\sin\theta_2\cos\theta_2\cos\theta_3 \tag{38b}$$

$$M_3 = (3GM/R^3)(I_1 - I_2)\sin\theta_3\cos\theta_3\cos^2\theta_2 \tag{38c}$$

For an Earth-pointing satellite, $\theta_1 = \theta_2 = \theta_3 = 0$. Stability conditions for this solution to Eqs. (36) and (37), when $k = 0$, have been discussed in Ref. 5 in terms of linear analysis and in Ref. 3 by including certain nonlinear effects. To study stability when $k \neq 0$, suppose temporarily that we have perfect measurements, that is $\dot{\bar\theta}_i = \dot\theta_i$ $(i = 1,2,3)$. Then, defining $K_1 = (I_2 - I_3)/I_1$ and $K_2 = (I_3 - I_1)/I_2$, and using $GM = \Omega^2 R^3$, one finds that the characteristic equation associated with the linearized variational equations corresponding to Eqs. (36) and (37) is

$$\left[\lambda^2 + k\lambda + \frac{3(K_1 + K_2)}{1 + K_1K_2}\right]\{\lambda^4 + 2k\lambda^3 + [k^2 + (1 + 3K_2 - K_1K_2)]\lambda^2 + k(K_2 - K_1)\lambda - 4K_1K_2\} = 0 \tag{39}$$

Hence, sufficient conditions for asymptotic stability are

$$\begin{gathered}(K_1 + K_2)/(1 + K_1K_2) > 0, \quad K_1K_2 < 0, \quad K_2 - K_1 > 0 \\ k > 0, \quad k^2 + 1 + 3K_2 - K_1K_2 > 0 \\ 2\{(K_2 - K_1)[k^2 + 1 + 3K_2 - K_1K_2] + 8K_1K_2\} - (K_2 - K_1)^2 > 0\end{gathered} \tag{40}$$

We will now show how $\dot{\bar\theta}_i$ $(i = 1,2,3)$ can be obtained by using two gyroscopes whose rotor spin-axes have different nominal orientations.

Let $\mathbf{m}_1$ and $\mathbf{m}_2$ be unit vectors parallel to the spin-axes, and define

$$\mathbf{m}_3 = \mathbf{m}_1 \times \mathbf{m}_2 \tag{41}$$

Next, note that the unit vectors $\mathbf{n}_i$, $\mathbf{e}_i$, $\mathbf{a}_i$ $(i = 1,2,3)$ are related to each other by

$$[\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3] = [\mathbf{n}_1\mathbf{n}_2\mathbf{n}_3][N] \tag{42}$$

and

$$[\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3] = [\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3][\Theta] \tag{43}$$

where $[N]$ and $[\Theta]$ are orthonormal matrices. $[N]$ depends solely on the orbital motion and is given by

$$[N] = \begin{bmatrix}\cos\Omega t & -\sin\Omega t & 0 \\ \sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 1\end{bmatrix} \tag{44}$$

$[\Theta]$ involves trigonometric functions of θ_1, θ_2, and θ_3 and is given by

$$[\Theta] = \begin{bmatrix}c_2c_3 & -c_2s_3 & s_2 \\ s_1s_2c_3 + s_3c_1 & -s_1s_2s_3 + c_3c_1 & -s_1c_2 \\ c_1s_2c_3 + s_3s_1 & c_1s_2s_3 + c_2s_1 & c_1c_2\end{bmatrix} \tag{45}$$

where $s_i = \sin\theta_i$, $c_i = \cos\theta_i$ $(i = 1,2,3)$. Similarly the unit vectors $\mathbf{m}_i$, $\mathbf{n}_i$ and $\mathbf{a}_i$ $(i = 1,2,3)$ are related by

$$\begin{gathered}[\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3] = [\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3][M] \\ [\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3] = [\mathbf{n}_1\mathbf{n}_2\mathbf{n}_3][C]\end{gathered} \tag{46}$$

where the elements of $[M]$ (and their derivatives) are to be determined by onboard measurements (e.g., of gimbal angles); whereas, those of $[C]$ are to be obtained from theoretical considerations, such as those presented in the previous section, which lead to an approximate description of the behavior of the spin-axis in inertial space.

An approximations to $[\Theta]$, to be denoted by $[\bar\Theta]$, can now be obtained by noting that the matrices $[N]$, $[\Theta]$, $[M]$, and $[C]$ are not independent of each other, since

$$\begin{gathered}[\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3] = [\mathbf{n}_1\mathbf{n}_2\mathbf{n}_3][N][\Theta] \\ [\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3] = [\mathbf{n}_1\mathbf{n}_2\mathbf{n}_3][C][M]\end{gathered} \tag{47}$$

Therefore,

$$[N][\Theta] \approx [C][M] \tag{48}$$

and, if $[\bar\Theta]$ is defined as

$$[\bar\Theta] = [N]^T[C][M] \tag{49}$$

then

$$[\Theta] \approx [\bar\Theta] \tag{50}$$

We will employ two identical gyroscopes having inertia properties such that $J_2 = 0$, and start both in such a way that $\gamma_O = 0$. As pointed out previously, this will insure that $\Delta = 0$.

The first gyroscope will be positioned in the satellite in such a way that $\beta_O + \alpha_O = 3\pi/2$; whereas, the second gyroscope will be positioned such that $\beta_O + \alpha_O = 0$.

Using Eqs. (16), (18), (20), (22), (27), (34), and (41) one now finds that

$$\mathbf{m}_1 = \mathbf{n}_1 + \tilde{\rho}_1\mathbf{n}_2 + \tilde{\rho}_2\mathbf{n}_3 \tag{51a}$$

$$\mathbf{m}_2 = \tilde{\rho}_1\mathbf{n}_1 + \mathbf{n}_3 - \tilde{\rho}_2\mathbf{n}_3 \tag{51b}$$

$$\mathbf{m}_3 = -\tilde{\rho}_2\mathbf{n}_1 + \tilde{\rho}_2\mathbf{n}_2 + \mathbf{n}_3 \tag{51c}$$

where $\tilde{\rho}_p$ and $\tilde{\rho}_2$ are defined as

$$\tilde{\rho}_1 = (\tilde{h}_4\tilde{k}_1/\tilde{p})\sin\tilde{p}t - (1/2\Omega)(\tilde{h}_3\Omega + [\tilde{h}_4\tilde{h}_6/(\tilde{p}^2 - 4\Omega^2)]\sin 2\Omega t \tag{52}$$

$$\tilde{\rho}_2 = (\tilde{h}_5/\tilde{p}^2) + \tilde{k}_1\cos\tilde{p}t - (\tilde{h}_6/\tilde{p}^2 - 4\Omega^2)\cos 2\Omega t$$

with $\tilde{h}_3$, $\tilde{h}_4$, $\tilde{h}_5$, $\tilde{h}_6$, $\tilde{p}$, and $\tilde{k}_1$ given by

$$\tilde{h}_3 = \tfrac{3}{4}(J_4/J_1);\ \tilde{h}_4 = -(J_3/J_1)\sigma;\ \tilde{h}_5 = \tfrac{3}{4}(J_3J_4/J_1J_5)\sigma\Omega \tag{53}$$

$$\tilde{h}_6 = \tfrac{3}{4}(J_3J_4/J_1J_5)\sigma\Omega,\ \tilde{p} = [J_3/(J_1J_5)^{1/2}]\sigma;$$

$$\tilde{k}_1 = (J_4/J_3)(\Omega/\sigma)^3\, 3/[J_3^2/J_1J_5 - 4(\Omega/\sigma)^2]$$

The matrix $[C]$ can thus be expressed as [see Eqs. (46) and (51)]

$$[C] = \begin{bmatrix} 1 & \tilde{\rho}_1 & -\tilde{\rho}_2 \\ \tilde{\rho}_1 & 1 & \tilde{\rho}_2 \\ \tilde{\rho}_2 & -\tilde{\rho}_2 & 1 \end{bmatrix} \tag{54}$$

For small angles θ_1, θ_2, and θ_3,

$$[\Theta] = \begin{bmatrix} 1 & -\theta_3 & \theta_2 \\ \theta_3 & 1 & -\theta_1 \\ -\theta_2 & \theta_1 & 1 \end{bmatrix} \tag{55}$$

Letting $\mathring{\theta}_i$ denote the approximations to $\dot{\theta}_i$, $i = 1,2,3$, obtained by differentiating Eq. (50), one finds by substitution from Eqs. (44), (54), and (55) into Eq. (49) that

$$\mathring{\theta}_1 = \dot{\tilde{\rho}}_2M_{12} - \dot{\tilde{\rho}}_2M_{22} + \dot{M}_{32} + \tilde{\rho}_2\dot{M}_{12} - \tilde{\rho}_2\dot{M}_{22} \tag{56a}$$

$$\mathring{\theta}_2 = -\dot{\tilde{\rho}}_2M_{13} + \dot{\tilde{\rho}}_2M_{23} - \dot{M}_{33} - \tilde{\rho}_2\dot{M}_{13} + \rho_2\dot{M}_{23} \tag{56b}$$

$$\begin{aligned}\mathring{\theta}_3 = {} & (\dot{\tilde{\rho}}_1 - \Omega)\cos\Omega t - \tilde{\rho}_1\Omega\sin\Omega t M_{11} + \\ & [-(\dot{\tilde{\rho}}_1 + \Omega)\sin\Omega t - \tilde{\rho}_1\Omega\cos\Omega t]M_{21} + \\ & [(\dot{\tilde{\rho}}_1 - \Omega)\sin\Omega t + (\dot{\tilde{\rho}}_2 + \Omega)\cos\Omega t]M_{31} + \\ & (-\sin\Omega t + \tilde{\rho}_1\cos\Omega t)\dot{M}_{11} + (\cos\Omega t - \tilde{\rho}_1\sin\Omega t)\dot{M}_{21} + \\ & \tilde{\rho}_2(\sin\Omega t + \cos\Omega t)\dot{M}_{31}\end{aligned} \tag{56c}$$

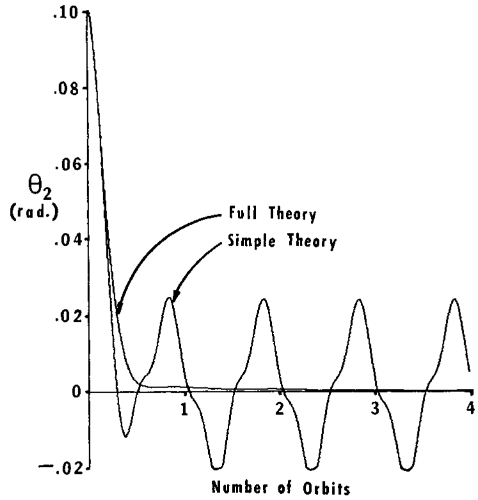

Fig. 9 Roll angle.

where M_{ij} $(i = 1,2,3)$ are the elements of $[M]$, $\dot{M}_{ij}$ their time derivatives, and $\dot{\tilde{\rho}}_1$ and $\dot{\tilde{\rho}}_2$ are given by

$$\dot{\tilde{\rho}}_1 = \tilde{h}_4\tilde{k}_1\cos\tilde{p}t - [\tilde{h}_3\Omega + (\tilde{h}_4\tilde{h}_6/(\tilde{p}^2 - 4\Omega^2)]\cos 2\Omega t \tag{57}$$

$$\dot{\tilde{\rho}}_2 = -\tilde{k}_1\tilde{p}\sin\tilde{p}t + [2\tilde{h}_6\Omega/(\tilde{p}^2 - 4\Omega^2)]\sin 2\Omega t$$

To summarize, attitude control of an Earth-pointing satellite can be achieved as follows: For a satellite that meets the requirements imposed by the inequalities (40), select, place, and start two gyroscopes in such a way that $J_2 = 0$, $\gamma_O = 0$, $\beta_O + \alpha_O = 3\pi/2$, and $\beta_O + \alpha_O = 0$; find the elements of $[M]$ [see the first of Eqs. (46)] and the time derivatives of these elements by measuring gimbal angles; evaluate $\mathring{\theta}_1$, $\mathring{\theta}_2$, and $\mathring{\theta}_3$ by means of Eqs. (56); and apply a control torque in accordance with Eq. (35).

Figures 8–10 show the behavior of θ_1, θ_2, and θ_3 for a satellite such that $K_1 = -0.2$, $K_2 = 0.5$, these being typical values for which inequalities (40) are satisfied. The initial conditions used to generate these curves are $\theta_{10} = \theta_{20} = \theta_{30} = 0.1$, $\dot{\theta}_{10} = \dot{\theta}_{20} = \dot{\theta}_{30} = 0$. The constant k is chosen such that the linearized differential equation governing the pitch angle θ_3 has a critically damped solution; that is $k = 2[3(K_1 + K_2)/(1 + K_1K_2)]^{1/2}$. The inertia properties of the gyroscopes are those given in Eqs. (32).

The two curves in each of Figs. 8–10 were obtained by means of numerical solutions of Eq. (36) and (37). The full theory curves resulted from using Eqs. (56) to evaluate $\mathring{\theta}_i$, $i = 1,2,3$, and assuming that the spin-axis directions of the

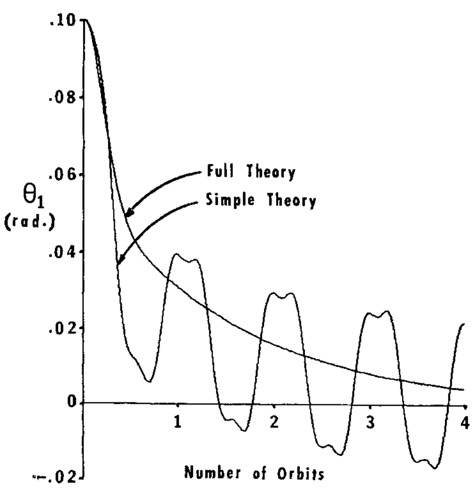

Fig. 8 Yaw angle.

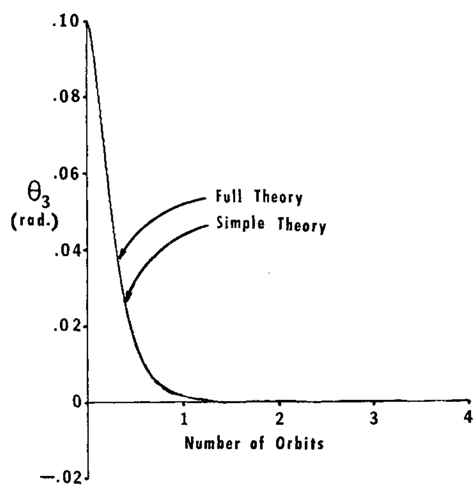

Fig. 10 Pitch angle.

gyro rotors are governed by Eqs. (51); whereas, the simple theory curves were generated by assuming that the spin-axis directions remain fixed in-inertial space, that is, $\mathbf{m}_i = \mathbf{n}_i$ $(i = 1,2,3)$.

It is seen that the full theory accomplishes its intended purpose; that is, θ_1, θ_2, and θ_3 approach zero asymptotically. Use of the simple theory, on the other hand, leads to oscillatory behavior of θ_1 and θ_2. Since oscillatory behavior of θ_1, θ_2, and θ_3 occurs in the absence of any control torque, it thus appears that the use of simple theory can produce pitch control, but is of little value as regards roll and yaw control. For a control scheme of the kind under consideration, the use of full theory therefore leads to significant improvements in performance.

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Thrust Profile Shaping for Spin-Stabilized Vehicles

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Analytical results are presented which show that the wobble motion and average pointing error caused by large thrust-line offsets can be greatly reduced by employing a shaped rocket thrust profile. A simplified analytical procedure is developed to evaluate the effect of thrust shape on the wobble motion and the average pointing error during the thrust and post-thrust periods. Performance relative to a nonshaped rectangular profile is conveniently expressed in terms of *K*-factors which depend on the Laplace transform of the shaped profile. This formulation shows that the relative pointing error *K*-factor is of the same form as the wobble motion *K*-factor with the spin rate ω replacing the polhode rate Ω. Thrust rise and decay shaping effects are presented for elementary time functions in terms of generalized parameters. The results provide a simplified basis for the preliminary specification of thrust shaping requirements and for the evaluation of candidate rocket motor designs.

Nomenclature

x, y, z	= principal body axes (x-axis longitudinal)
ω, q, r	= angular velocities about x, y, and z axes, respectively
I_x, I	= moments of inertia about x-axis and transverse axes (y, z), respectively
ϕ, θ, ψ	= roll, pitch, yaw Euler angles of body axes with respect to inertial axes
Ω	= $(I - I_x)\omega/I$, polhode rate
ω_{yz}	= $q + ir$, transverse rate
α	= $\theta + i\psi$, inertial pointing angle
$N(t)$	= disturbance torque about z-axis
N	= maximum value of $N(t)$
$n(t)$	= $N(t)/N$
β	= frequency of sinusoidal thrust profiles
$K_{\tilde{\omega}_{yz}}, K_{\tilde{\alpha}}$	= thrust shaping performance factors for oscillatory transverse rates and average pointing errors, respectively (relative to a rectangular profile)

Subscripts

o, f	= evaluated at thrust onset, thrust termination, respectively
r, d	= evaluated at end of thrust rise, beginning of thrust decay, respectively

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Introduction

SPIN stabilization performance during vacuum thrusting maneuvers has generally been analyzed for constant body-fixed torques corresponding to a nonshaped rectangular thrust-time profile. The interaction between thrust shape and spin dynamics appears to have received little attention in the literature.[1–7] Papis[1] investigated the interaction for a ramp input based upon a concern about the discrepancy between the finite thrust rise-time of actual rocket motors and the instantaneous rise of the rectangular thrust model.

The concept of deliberately shaping the thrust profile in order to enhance spin stabilization performance evolved from applications with a large thrust-line offset and a small difference between the roll and transverse inertias. The wobble motion (oscillatory transverse rate) caused by the thrust-line offset during thrust rise and decay is inversely proportional to the product of the inertia difference $I - I_x$ and the spin rate, ω. Large wobble motions cause a loss in the imparted velocity increment (cosine loss) and may seriously degrade post-thrust attitude stability if the vehicle is de-spun to a low-roll rate after thrust completion. The average pointing error is less difficult to control in such applications since it is inversely proportional to the *square* of the spin rate. Thrust profile shaping via solid motor propellant grain design (or combinations of propellant segments with different burning rates) appears to be an attractive method of control for both the oscillatory transverse rate and the average pointing error. This paper presents an analytical basis for the design of a thrust-shaped spin stabilization system.